Flow Around A Cylinder

A common geometry in fluids engineering, as well as in many applications, is the flow of a fluid past a circular cylinder. Analysis of this problem has mathematical applications in conformal mapping for airfoil theory, as well as engineering applications (i.e. drag on a golf ball) and geo-physical modeling (i.e. weather patterns around large objects).

To begin our analysis of this problem, we start with the simplest model which concerns flow around the cylinder at low Reynolds numbers. Reynolds number, which is dimensionless and plays an important role in all viscous flows, is:

$$\mathrm{Re}_L = \frac{\rho U L}{\mu}$$

Where U is a velocity, L is a characteristic geometric size and rho and mu are the fluid density and viscosity, respectively. For the cylinder problem, we can recast this as:

$$\mathrm{Re} = \frac{Ud}{\nu}$$

Where d is the diameter of the cylinder and nu is the kinematic viscosity of the fluid. The flow field over the cylinder will be symmetric at low values of Reynolds numbers. As the Reynolds number increases, the flow pattern changes from smooth or laminar, through a transitional region into the fluctuating or turbulent regime.

For the cylinder case, as Reynolds number is increased, flow begins to separate behind the cylinder causing vortex shedding, a phenomenon called Karman vortex street, which is an unsteady phenomenon.

We can state the following assumptions, which allow us to simplify the governing flow equations to a set of analytical equations.

Assumption 1: 2-D flow

> The assumption is made that this is only a 2-dimension problem and that there is no z-depth

Assumption 2: Incompressible flow

> Incompressible flow is valid for low speed air flows and for many flows involving liquids

Assumption 2: Inviscid flow

Since we are examining low speed flows past the cylinder, we can assume no viscous effects, which will allow the problem to simplify to an analytic solution

In undergraduate aerodynamics, it is taught that the flow resulting from the superposition of incompressible, irrotational flows is also incompressible and irrotational. To model the incompressible, irrotational flow about a circular cylinder in a uniform stream, we will superimpose a uniform flow and a doublet. Solving for the analytic equation, in polar coordinates, the solution will simplify to:

$$u_r = U_\infty (1 - \frac{a^2}{r^2}) \cos(\theta) \qquad (r \geq a)$$

$$u_\theta = -U_\infty (1 + \frac{a^2}{r^2}) \sin(\theta) \qquad (r \geq a)$$

Where a is the radius of the cylinder and r is the radial distance to any point in the flowfield from the center of the cylinder.

To calculate the ideal flow around a cylinder, we will use a Java applet hosted at the the “Java Applets for Engineering Education” web site, which was an NSF funded project at Virginia Tech. This site hosts several Java Applets that connect calculations with data visualization, to allow students to interact with the science phenomenon. Applets have been developed for fluid dynamics, statics and dynamics.

The URL is:

http://www.engapplets.vt.edu/

For our first look at the ideal flow around a cylinder, we will use the Ideal Flow Machine. After going to the engAPPLET web page, click on Ideal Flow Machine, which is listed in the left hand column, as seen in Figure 1.

You will see a page that contains links to the Java source code, instructions as well as examples. After Java loads, a box will appear with the title “Launch Ideal Flow Machine”. Click this box when it appears.

When the Ideal Flow Machine appears, click on the pull down menu in the upper left hand corner and select “Free Stream”, as shown in Figure 2. Click anywhere on the mesh. An arrow will appear on the left hand side of the screen, showing the free stream setting. The flowfield has now been initialized.

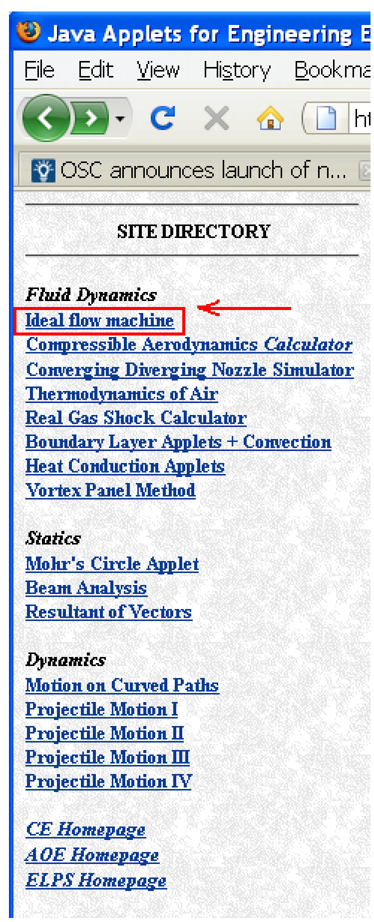

Figure 1

To create the geometry, click on the pull down menu again and select "Circle". Move the mouse pointer to the center cross hair (the x and y coordinates should read 0.0) and click on the crosshair, holding the left mouse button down. Drag the mouse in any direction to a neighboring crosshair, until the size of cylinder you want is set. You should see a green circle appear and change size, based on where you move the mouse. Release the left mouse button when your cylinder is the desired size.

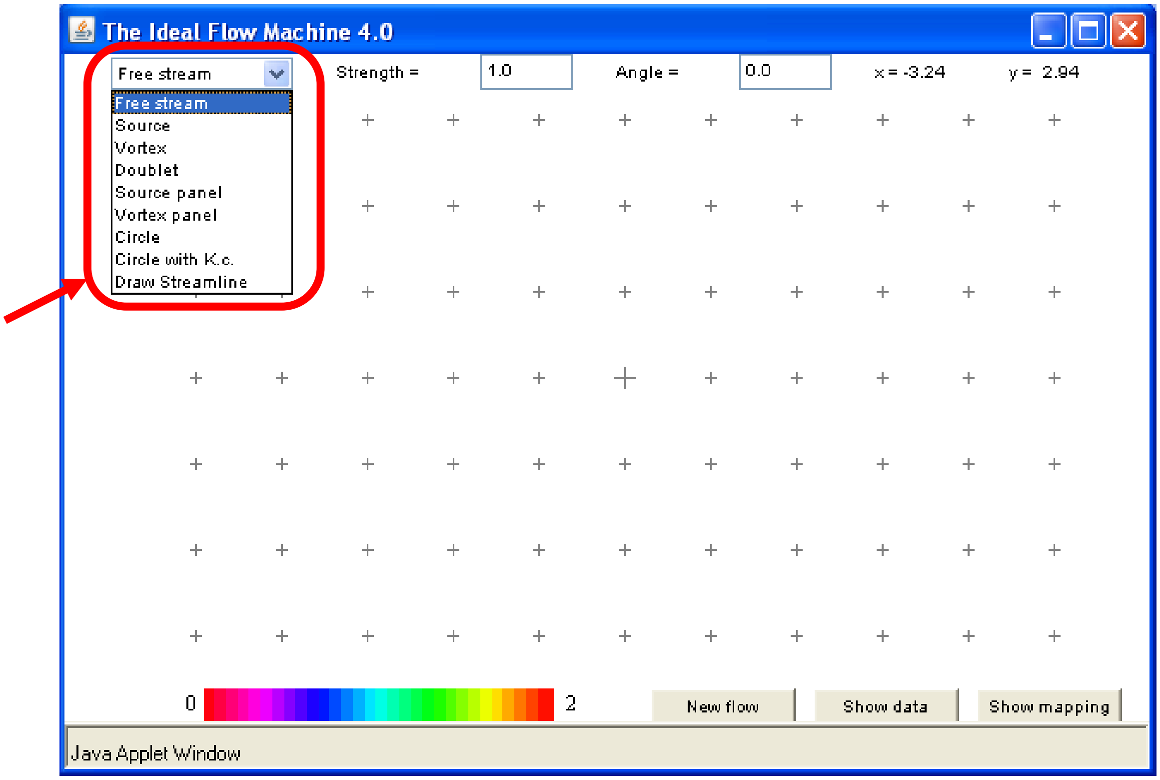

Figure 2

To set the doublet, select "doublet" from the pull down menu. Set the strength to 10.0 and the angle to -180.0 to set the proper orientation. Move the mouse pointer to the center cross hair (the x and y coordinates should read 0.0) and click the left mouse button to set the doublet.

To draw the streamlines, select "Draw Streamline" from the pull down menu. Once it has been selected, click anywhere on the mesh and you will see a streamline generated from that point.

After creating several streamlines, click on "Show Data" and look at the magnitude of the calculated horizontal velocities (variable u in the 3rd column). With higher freestream velocities, it would be expected to see some flow separation behind the cylinder, but the inviscid assumption prevents this.

Another example of how this simplification is not realistic occurs at the surface of the cylinder. Examine the 2 equations for velocity given earlier. At the surface of the cylinder we would expect the velocity to be zero, due to the no-slip condition imposed by intermolecular forces between the fluid and the solid. The equation for velocity in the radial direction satisfies this condition, but the tangential velocity component has a finite values when r = a, violating the no-slip condition.